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A Simplified Mathematical Model for the Motion of a Tethered Kite Balloon: Stability Criteria

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Abstract

Some simplifying assumptions and noticing properties of streamlined balloons lead us to a system of three equations with second degree derivatives, which is valid only for small motions. Predicting large motions needs the use of computing machines and a complete system of equations. Nevertheless the simplified model is adequate to derive stability criteria of the balloon. These criteria provide important relations between mechanical and aerodynamic parameters.

35.1 INTRODUCTION

This study has been done in order to give some evidence of the importance of some balloon parameters; especially, critical wind speed, mooring altitude, lateral lift coefficient, and rotational damping coefficient.

The model is simplified. The kite has been assumed to be rigid, weight of the kite-wire negligible, and plane motions (constant altitude, constant pitch, negligible roll). All the aerodynamic parameters (lateral force and moment) are supposed to

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be linear functions of yaw angle ϕ_O. The strain due to the kite-wire is supposed to be proportional to angle α between cable and the vertical plane containing wind vector $\vec{V}$.

All these assumptions are valid for small motions.

35.2 SYMBOLS AND DEFINITIONS

35.2.1 Properties of Streamlined Balloons

L	Length of the kite-wire
M	Total mass: balloon, payload, additional mass of air
I	Total inertia momentum, relative to the gravity center G
δ	Distance between verticals of mooring point A and gravity center G
V	Wind speed
C_x, C_y, C_z	Aerodynamic forces coefficients: drag, lateral lift, vertical lift. The axis system is relative to the wind.
F_x, F_y, F_z	Correspondent forces. Related to their coefficients by relations as the following one:

$$F_y = C_y \, \rho S \, \frac{V^2}{2}$$

where ρ is air density, and S the main cross section of the balloon.

C_n	Aerodynamic torque coefficient relative to the vertical of point A.
M_n	Moment of the torque. H being the balloon length:

$$M_n = C_n \, H \, \rho \, S \, \frac{V^2}{2}$$

ϕ	Projection on the horizontal plane of the yaw angle
α	Angle of the kite-wire relative to the vertical plane containing wind vector $\vec{V}$
F_v	Kite-wire tension. Due to bouyancy and aerodynamic vertical lift plus drag
F_ϕ	Derivative of F_y force relative to angle ϕ:

$$F_\phi = \frac{\partial F_y}{\partial \phi}$$

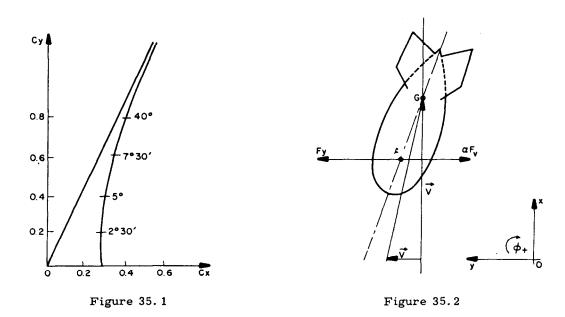|	
M_ϕ	Derivative of the moment M_n relative to angle ϕ:

$$M = \frac{\partial M_n}{\partial \phi}$$

R_ϕ Rotation damping coefficient relative to the gravity center G.

We supposed first that kite-balloon notion was plane (ϕ and α supposed to be small). Some aerodynamic considerations lead us to assume this motion linear.

If yaw angle increases from 0° to 10°C, the C_x increment is 0.1 (0.3 to 0.4) when C_y increases from 0.0 to 0.8 (see Figure 35.1).

The balloon being initially in equilibrium with a yaw angle of 0°, a crosswind nonequilibrated force F_y will appear, and point A will move along C_y axis. The kite-wire will provide a strain αF_v (see Figure 35.2), which is oriented along axis O_y.

Thus the motion can be described with angles α and ϕ only.

Figure 35.1

Figure 35.2

35.3 EQUATIONS OF THE MOTION

If the balloon moves with a speed v, wind speed being $\vec{V}$, the aerodynamic forces will be due to relative wind vector $\vec{V} - \vec{v}$. This vector has a ϕ_0 angle with the balloon axis (apparent yaw angle). Let us now examine the forces and torques system relative to gravity center G.

Forces are:

F_y

$- \alpha F_V$

Torques are:

$$- M_n + \delta (F_y - \alpha F_v)$$

$$- g\left(\frac{d\phi_o}{dt}\right).$$

This second torque is the damping torque, due to rotation speed $\frac{d\phi_o}{dt}$. Wind tunnel tests and theoretical calculations showed that g is a linear function of $\frac{d\phi_o}{dt}$. Thus we shall write:

$$g\left(\frac{d\phi_o}{dt}\right) = R_\phi \frac{d\phi_o}{dt}.$$

The following equations describe the forces and torques system relative to point G;

$$\left|
\begin{array}{l}
ML \dfrac{d^2\alpha}{dt^2} = -F_v \alpha + F_\phi \phi_o + M\delta \dfrac{d^2\phi}{dt^2} \\[2em]
I \dfrac{d^2\phi}{dt^2} = (-F_v \alpha + F_\phi \phi_o)\delta - M_\phi \phi_o - R_\phi \dfrac{d\phi_o}{dt} \\[2em]
\phi_o = \phi - \dfrac{L}{V} \dfrac{d\alpha}{dt} + \dfrac{\delta}{V} \dfrac{d\phi}{dt}.
\end{array}
\right.$$

35.4 LAPLACE TRANSFORM OF THE SYSTEM

The initial assumptions we did lead us to have

$$F_\phi$$

$$M_\phi$$

which are the derivatives relative to ϕ of

$$F_y$$

$$M_n$$

as constants. Thus, the former system of mechanical equations can be treated by LAPLACE transform:

α becomes γ_1

ϕ becomes γ_2

ϕ_o becomes γ_3

As we need to derive criteria stability, we will suppose wind vector $\vec{V}$ to be constant and the balloon in initial equilibrium. α, ϕ, and their first and second order derivatives relative to the time are zero for time zero.

Such LAPLACE transform can be written as:

$$\left|
\begin{array}{l}
0 = \gamma_1 \left[- MLs^2 - F_v \right] + \gamma_2 \, M\delta s^2 + \gamma_3 \, F_\phi \\[2ex]
0 = \gamma_1 \left[- \delta F_v \right] + \gamma_2 \left[- Is^2 \right] + \gamma_3 \left[\delta \, F_\phi - M_\phi - R_\phi s \right] \\[2ex]
0 = \gamma_1 \left[- \dfrac{L}{V} s \right] + \gamma_2 \left[1 + \dfrac{\delta}{V} s \right] - \gamma_3
\end{array}
\right.$$

These equations give a stable mechanical system if the roots of their coefficients determinant have positive real parts. Developing this determinant Δ gives a fourth degree expression:

$$\Delta = a_0 \, s^4 + a_1 \, s^3 + a_2 \, s^2 + a_3 s + a_4$$

with

$$a_0 = 1$$

$$a_1 = \frac{R_\phi}{I} + \frac{F_\phi}{MV}$$

$$a_2 = \frac{F_v}{ML} \; \frac{I + M\delta^2}{I} + \frac{M_\phi - \delta F_\phi}{I} + \frac{R_\phi}{IMLV} \quad \delta F_v$$

$$a_3 = \frac{F_v}{ML} \left(\frac{R_\phi}{I} + \frac{\delta M_\phi}{IV} \right)$$

$$a_4 = \frac{F_v \, M_\phi}{IML} \quad .$$

35.5 STABILITY CRITERIA

Respect of the former condition is given by ROUTH criteria. The three following conditions must be true:

(1) $a_0, a_1, a_2, a_3, a_4 > 0$

(2) $a_2 - \dfrac{a_0 \, a_3}{a_1} > 0$

$$(3) \quad a_3 - \frac{a_1 a_4}{a_2 - \frac{a_o a_3}{a_1}} > 0$$

In fact, mechanical parameters are such that only the term a_2 can be negative due to the expression

$$M_\phi - \delta F_\phi.$$

This expression represents the slope of aerodynamic momentum torque curve relative to the gravity center, majored of the term δF_x. This slope would be positive if the aerodynamic transversal force was located behind gravity center. Anyway we can conclude that drag increases stability.

The second condition is more restraining than the first one if damping coefficient R_ϕ or torque momentum coefficient M_ϕ (relative to the mooring point) are too important.

The third condition assigns a minimum altitude of stability.

35.6 CONCLUSIONS

For a given altitude, classical balloons have a critical wind speed. Even with streamlined wind, they move in a crosswind direction. At the same altitude, homotetical balloons have a critical wind speed proportional to the volume square root.

For a given wind speed, classical balloons have two critical altitudes of stability. If a balloon has a considerable kite effect or aerodynamic resultant behind gravity center, it will have only a minimum height of stability and no critical speed.

Some other conclusions are surprising. For instance, drag always has an improving effect. Thus, Crude-Section fins (like Caquot fins for instance) are more efficient than streamlined ones of the same shape. And streamlined hulls, class C-like can be more difficult to stabilize than cruder ones.

Lastly, to stabilize balloons it is necessary to have fins providing an important torque, less lateral lift, much vertical lift.

Thus vertical fins must be small, in a very rear position. Horizontal fins must be large in a middle position. This is possible, as far as vertical fins are concerned, if the balloon is small. Upon large ships such fins would bend the hull.

One of the solutions we experimented successfully with was to fit a parachute upon the upper fins. This device increased drag and provided a damping coefficient ten times higher. Critical wind speed was thus raised. Over critical speed, the movement amplitude was reduced by damping.